IV. SOLID STATE PHYSICS

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A. ELASTIC CONSTANTS OF SODIUM IODIDE

The three adiabatic elastic constants of sodium iodide were measured from 170° K to 300° K by an ultrasonic pulse-echo technique. The constants c_{11} and c_{44} were obtained from velocity measurements in the [100] direction by use of the relations

$$\rho U_\ell^2 = c_{11} \qquad \text{and} \qquad \rho U_t^2 = c_{44}$$

To obtain c_{12}, measurements were made with longitudinal waves in the [110] direction, for which

$$\rho U_\ell^2 = (c_{11} + c_{12} + 2c_{44})/2$$

Measurements were not made below 170° K because of failure of the seal between sample and quartz transducer, and spontaneous cleaving of the sample along the [100] faces upon further cooling.

The results are given in Fig. IV-1. The curves for c_{11} and c_{44} were extrapolated

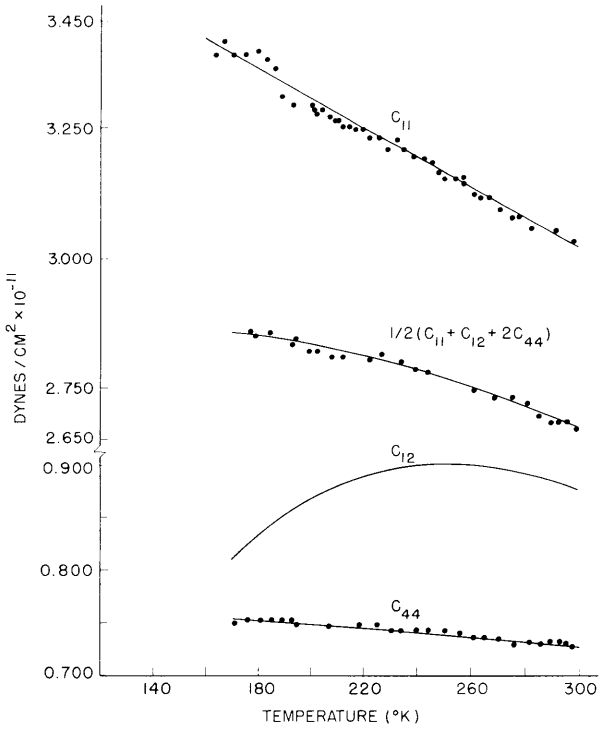

Fig. IV-1. Elastic constants of sodium iodide versus temperature.

to 0° K. It was not possible to extrapolate c_{12} because of its large uncertainty; therefore c_{12} was assigned a -2 per cent variation from 300° K to 0° K based on the observed change for sodium chloride (1). The elastic constants extrapolated to 0° K are: $c_{11} = 3.711$, $c_{44} = 0.777$, and $c_{12} = 0.857$, all in units of 10^{11} dynes/cm^2. With these 0° K values of the c_{ij}, a Debye characteristic temperature, θ_o, of 167° K was calculated. The calorimetric value of θ_o reported by Morrison (2) is 165° K.

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References

1. W. C. Overton and R. T. Swim, Phys. Rev. 84, 758 (1951).
2. W. T. Berg and J. A. Morrison, Proc. Roy. Soc. (London) A242, 467 (1957).

B. RECOMBINATION OF ELECTRONS WITH DONORS

1. Recombination

In the Quarterly Progress Report of January 15, 1958, page 25, we published some curves for recombination of electrons with donors which were plotted as straight lines on log-log paper. The assumption that was made to explain this experimental behavior was that the times involved were short compared with the time constant τ that appears in the solution of the differential equation describing the decay of the density:

$$\frac{dn}{dt} = -\alpha n(n + N_A) + \beta(N_D - N_A - n) + \gamma(N_D - N_A - n)n \qquad (1)$$

The solution of Eq. 1 with $\gamma = 0$ is

$$n - n_\infty = \frac{1}{\alpha\tau} \frac{1}{\left(\frac{1}{N_o \alpha\tau} + 1\right) e^{t/\tau} - 1}$$

where n_∞ is the density at time $t = \infty$ when $dn/dt = 0$; N_o is the initial density at time $t = 0$; $\tau^{-1} = (N_A + 2n_\infty)\alpha + \beta$; and α is the probability of recombination, β of thermal ionization and γ of impact ionization. If we extrapolate toward time $t = 0$, the exponential part of the solution, its intercept with the time axis, is

$$(n - n_\infty)_{ext} = \frac{1}{\alpha\tau} \frac{1}{\frac{1}{N_o \alpha\tau} + 1} = \frac{N_o}{1 + N_o \alpha\tau}$$

Since $\tau^{-1} \approx \alpha N_A$, we obtain

$$(n - n_\infty)_{ext} \approx \frac{N_o N_A}{N_A + N_o} \approx N_A$$

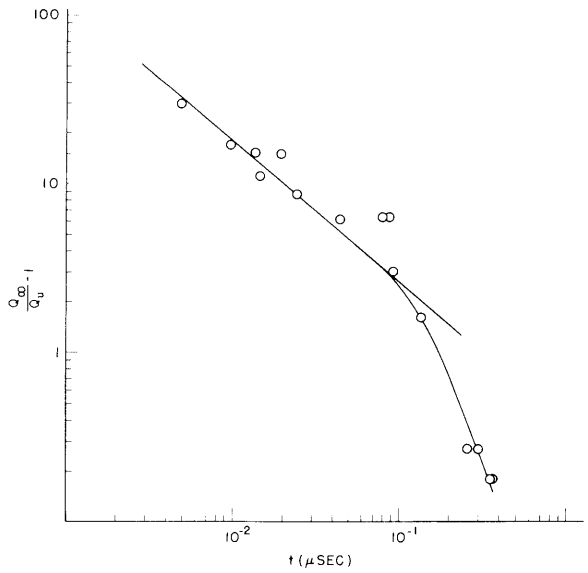

Fig. IV-2. Decay of electron density as a function of time. Sample 3.

which permits the determination of N_A.

The assumption that $t \gg \tau$, however, is not satisfied by our earlier curves, and this is the reason for the deviation of the points in Fig. IV-2 from the straight line. If we replot on a semi-log plot the data of Fig. IV-2 and other data from some samples of the same crystal which were not included in the earlier report, we have the data shown in Fig. IV-3. The Q of the cavity was different for each series of measurements (each series refers to the sample in a slightly different position in the cavity or to different samples); the curves were found to be displaced parallel to each other. Hence, to plot them uniformly, we chose the point indicated by the arrow in Fig. IV-3 and scaled all the values of $(Q_\infty)/(Q_u) - 1$ so that they would coincide at that point. (This scaling is essentially equivalent to changing the losses of the cavity.) The time constant was measured as $6.5\ 10^{-8}$ sec. We cannot, however, say anything about the compensation because the $(Q_\infty)/(Q_u) - 1$ scale was not calibrated for densities, and we do not know the type of impurities that are present (that is, whether they are As, Sb, P or Li).

With the help of Dr. Varnerin, of Bell Telephone Laboratories, Incorporated, we received a sample of germanium doped with antimony which has a room temperature resistivity of 30 ohm-cm and a liquid-nitrogen resistivity of 5.5 ohm-cm, which corresponds to $N_D - N_A = 3.8\ 10^{13}/cm^3$. The compensation was evaluated as less than 10 per cent and more than 1 per cent.

The results for this sample are shown on Fig. IV-4. Here, since we know the Q of the cavity when the sample is not broken down, we can calibrate the losses in the sample,

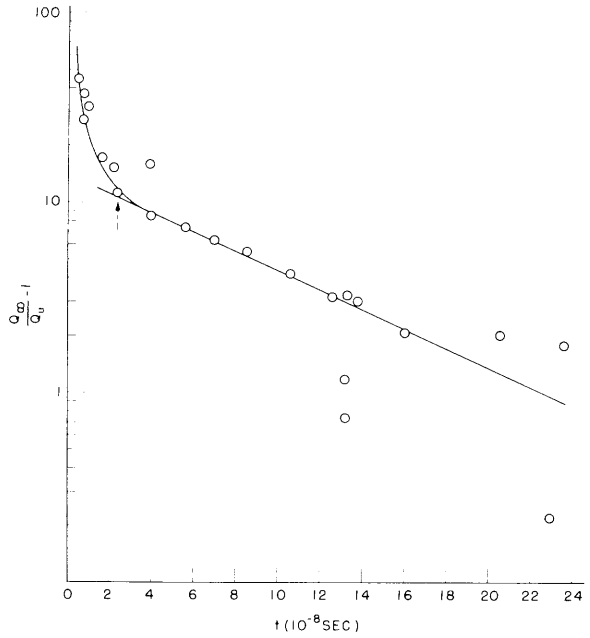

Fig. IV-3. Decay of electron density as a function of time. Samples 1 and 3. The data are normalized to the point marked by the arrow.

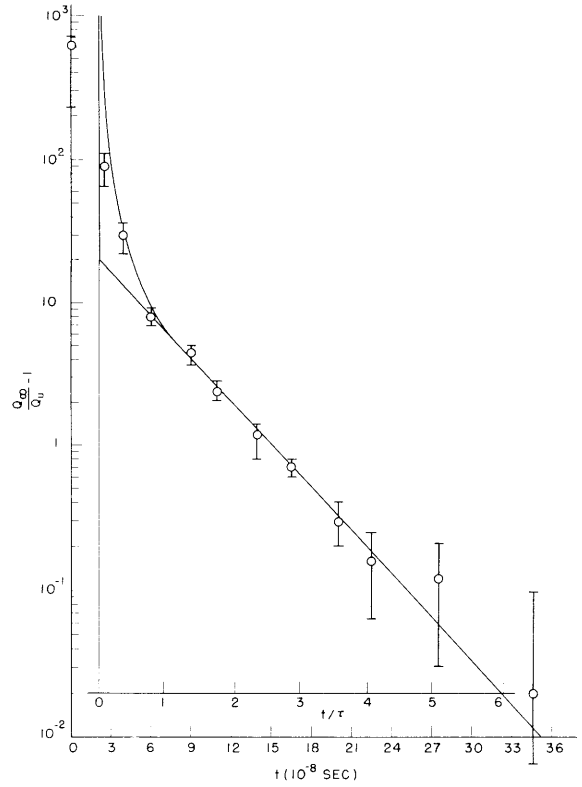

Fig. IV-4. Decay of electron density as a function of time. Sample BTL-1. The theoretical curve is in accordance with the solution of Eq. 1.

$$\frac{Q_\infty}{Q_{sample}} = \frac{Q_\infty}{Q_u} - 1$$

in terms of density of electrons. The microwave electric field was sufficiently low to ensure that there was no partial breakdown. The losses in the sample that was not broken down can be neglected, and we can write the Q of the cavity as losses that are all attributable to the sample:

$$\frac{1}{Q_{sample}} = \frac{1}{\omega\epsilon_o\rho} \frac{\int_{sample} E^2 dV}{\int_{cavity} E^2 dV}$$

and for the frequency shift, we have

$$\frac{\Delta\lambda}{\lambda} = \frac{K-1}{2} \frac{\int_{sample} E^2 dV}{\int_{cavity} E^2 dV}$$

The ratio of the two integrals can be measured, since we know the frequency shift of the cavity when the sample is introduced, but an assumption will have to be made for the mobility of the electrons which appears in the expression for the resistivity. If we assume that only ionized impurity scattering is acting, from the Brooks-Herring formula with $N_A = 10^{12}/cm^3$, we obtain $\mu = 2\ 10^5\ cm^2/volt\text{-}sec$. But the importance of neutral impurity scattering should not be overlooked; hence with a rather arbitrary criterion we take $\mu = 10^5\ cm^2/volt\text{-}sec$. This value for the mobility is probably the largest source of error in the evaluation of the cross section. With this value of mobility we obtain $N_A = 4.4\ 10^{11}/cm^{-3}$, which corresponds to $N_A/N_D = 1.15$ per cent. The observed time constant is related to the cross section for recombination by the expression

$$\tau = \frac{1}{N_A \langle \sigma v \rangle} \qquad (2)$$

We do not know the energy dependence of σ. Thus, even if we assume a Maxwellian distribution of the electrons, we cannot evaluate the average in Eq. 2. If we assume that

$$\tau = \frac{1}{N_A \sigma v}$$

then

$$\sigma = \frac{1}{\tau N_A v}$$

where $v = \left(\frac{3kT}{m^*}\right)^{1/2}$, and $m^* = m_o/4$. The value of the cross section which we obtain, $\sigma = 1.6\ 10^{-11}$ cm^2 at 4.2° K, is, however, approximately 16 times larger than that measured by Koenig (1).

2. Breakdown

We have continued to study the delay that is necessary for breakdown as a function of overvoltage. The breakdown field for sample BTL-1 is 4.9 volts/cm, with both dc and very long pulses. The interesting thing to notice is that the logarithm of the overvoltage and the logarithm of the time that is necessary for breakdown fall on a straight line. We do not have any good explanation for this phenomenon.

We also studied the breakdown of sample BTL-1 in the same way as we studied recombination, with a field of 40 volts/cm and a field of 18.2 volts/cm (see Fig. IV-5).

The time constant leading to equilibrium with the sample broken down could not be clearly followed in the fastest breakdowns (~ 40 volts/cm) and we can only give $\tau' \approx 1.4\ 10^{-8}$ sec as an upper limit for its value. With the slower breakdown (18.2 volts/cm), we obtain $\tau' = 16.2\ 10^{-8}$ sec (see Fig. IV-6).

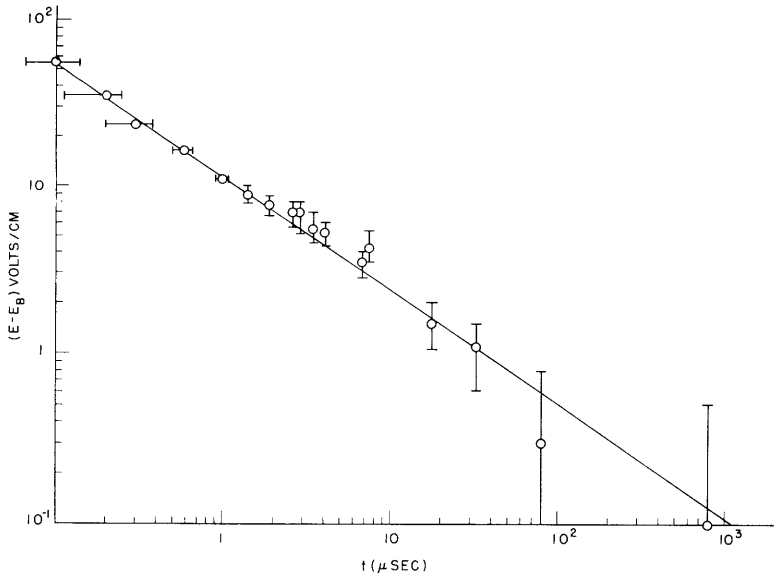

Fig. IV-5. Beginning of breakdown as a function of overvoltage. Sample BTL-1. Breakdown field, 4.9 volts/cm.

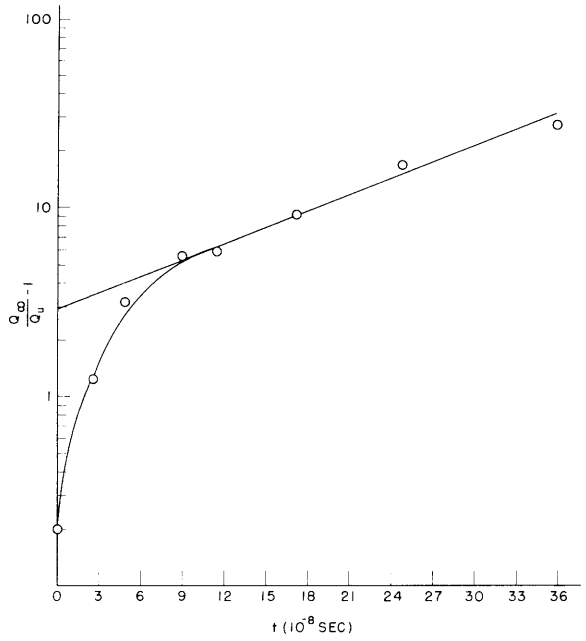

Fig. IV-6. Increase of electron density as a function of time at the beginning of breakdown. Sample BTL-1. Applied field, 18.2 volts/cm.

In the approximation made for recombination, the equation describing breakdown (Eq. 1) has the following solution when γ is not zero.

$$\frac{1}{n - n_\infty} = (\alpha + \gamma)\tau - \frac{3n_\infty - (N_D - N_A)}{n_\infty(2n_\infty - N_D - N_A)} e^{t/\tau}$$

where $\tau^{-1} = \gamma(2n_\infty + N_A - N_D) + \alpha(N_A + 2n_\infty) + \beta$, and the most important term is $\gamma(2n_\infty + N_A - N_D)$. With this approximation we shall have breakdown only when $n_\infty > (N_D - N_A)/2$, and consequently $\tau > 0$.

This solution of the differential equation allows us to calculate γ, but we cannot calculate σ, as we did in the case of recombination, because a small change in the electric field will cause an enormous change in $\gamma = \langle \sigma_i v \rangle$, even if σ_i is not energy-sensitive, since we greatly increase the number of electrons that have sufficient energy to ionize.

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1. S. H. Koenig, Hot and warm electrons – A review, International Conference on Semiconductors, University of Rochester, Sept. 18-22, 1958.

C. AN APPLICATION OF CYCLOTRON RESONANCE TO THE STUDY OF COLLISION TIMES AND DENSITIES OF CARRIERS AT LOW TEMPERATURES IN GERMANIUM

In n-type germanium the total conductivity σ can be written (1) as the sum of the conductivities of four different valleys:

$$\sigma = \sum_{n=1}^{4} \sigma_n \tag{1}$$

In a microwave field the σ_n have real and imaginary parts, and Eq. 1 separates into real and imaginary parts:

$$\sigma = \sigma_r + i\sigma_i \qquad \sigma_n = \sigma_{nr} + i\sigma_{ni}$$

$$\sigma_r = \sum_{n=1}^{4} \sigma_{nr} \qquad \sigma_i = \sum_{n=1}^{4} \sigma_{ni} \tag{2}$$

At the cyclotron resonance (2) for a given valley, $B_c = (m^*/e)\,\omega$ when $\omega\tau \gg 1$, we have

$$\sigma_r \approx \sigma_{4r} \tag{3}$$

Also at a value of magnetic field (2) near B_c, given by

$$B = B_c \left(1 + \frac{1}{2} \frac{1}{\frac{e^2}{m^{*2}} B_c^2 \tau^2} \right)$$

we have

$$\sigma_i = \sum_{n=1}^{3} \sigma_{ni} \tag{4}$$

In the development of Eqs. 3 and 4, valley 4 corresponds to the valley at cyclotron resonance.

The experimental condition for these approximations to hold is that the microwave electric field be perpendicular to the dc magnetic field. Experiments for measuring

$$\sigma_i = \sum_{n=1}^{3} \sigma_{ni}$$

at cyclotron resonance are being performed at 23.5 kmc.

T. Higier

References

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